Number of Dimensions of the Universe

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ABSTRACT

Analysing the features of quaternions and generalysed quaternions the necessity of multiplication by 4 of the number of dimensions occured. 4 superdimensions, D=44 or D=104 dimensions and loop-like subdimensions have been introduced in purpose to unification all attitudes and explanation all phenomena. According to Witten's formula expanding and vanishing loops have been introduced. The expanding ones wind the Universe and the vanishing ones weave the continuity of the Universe.

I
Introduction

A. Einstein has introducted 4-dimensional spacetime in the works leading to the rise of general relativity, which was turn in thinking and glance the reality.
The addition ofourth dimension made possible the union of mass and curvature of spacetime. Since that time it has been understood that the number of dimensions may be bigger than 3.
In the similar way Kaluza and Klein [1,2] have postulated introduction 5th dimension in purpose to unite gravitation with electromagnetism. In the attitude the 5th dimension has been tolled into the loop.
E. Witten has postulated 11-dimensional space in the works concerning strings and superstrings [3]. Now he inclines to 1o-dimensional space [4].
M.J.Duff has discovered that D=11 is critical number of dimensions of supermembrane [5,6], but not all objects have critical number of dimensions.
Nothing strange that Julia and Cremmer have considered the introduction D=12 dimensional space [7].
Next J.Goldstone and others [8] have introduced D=26 dimensional space. They have analysed massless relativistic string

quantisation only of the independent component was possible when the first excited state was photon and when D=26 (components are $x^{\mu}(\sigma,t)$ functions.

The Author of this work has introducted in previous articles [9,10] D=8 dimensional space because of introduction the possibility that $v>c$ or v complex, which was implicated directly by Lorentz's transformation.
Next the contrary tendency to reduction of number of dimensions arises. It has obtained the most dramatic character in the work of A.Ashtekar and others [11] who consider again 4-dimensional spacetime, which is weaved into 2-dimensional loops.
Two fundamental questions arise: how the tendency to increasing of the number of dimensions with the tendency to reduction of this number can be reconciled if it is generally possible and how many dimensions has the Universe really.

II
The multiple of four

In purpose to lead further consideration we must analyse algebra of quaternions before.
Quaternion has the shape $x = x_o + ix_1 + jx_2 + hx_3$ with multiplication table

$$i^2 = j^2 = k^2 = -1$$

$$ij = -ji = k$$

$$jk = -kj = i$$

$$ki = -ik = j$$

$$x_o,\ x_1,\ x_2,\ x_3 \in R$$

Using the multiplication table of quaternions we obtain

$$x^2 = x_o^2 - x_1^2 - x_2^2 - x_3^2 + 2ix_1x_o + 2jx_2x_o + 2kx_3x_o \qquad (1)$$

It is seen that $x^2 \in R$ when $x_o = 0$

Now we introduce generalized quaternions and their multiplication table

$$x = x_o + e_1x_1 + e_2x_2 + e_3x_3$$

	1	e_1	e_2	e_3
1	1	e_1	e_2	e_3
e_1	e_1	n	e_3	ne_2
e_2	e_2	$-e_3$	m	$-m_1e_1$
e_3	e_3	$-ne_2$	me_1	$-nm$

$n, m \in Z$
$x_o, x_1, x_2, x_3 \in R$

It is seen that analogically like in the case of quaternions

$$x^2 = x_o^2 + nx_1^2 + mx_2^2 - nmx_3^2 + 2e_1x_ox_1 + 2e_2x_ox_2 + 2e_3x_ox_3 \qquad (2)$$

And again $x^2 \in R$ when $x_o=0$.

The structure of signs of vector product in the real part appears.
The signature of signs of the real part of quaternions is (+---) and of generalized quaternion is (+---) or (+-++) or (++-+) or (+++-).

It is easy seen, when:

$n > 0$ and $m > 0$ (+++-)

$n < 0$ and $m < 0$ (+---)

$n < 0$ and $m > 0$ (+-++)

$n > 0$ and $m < 0$ (+++-)

Such signature of the real part of second power of quaternion and generalized quaternion is analogical to signature of Minkowski's spacetime, what proves the possibility of use of quaternions and generalized quaternions to the description of the number of dimensions of the Universe.

2.2

It is easy to show that taking quaternions under consideration introduces the necessity of multiplication of all hypothetical numbers of dimensions of the Universe D by 4. Then the number of dimensions of the Universe is equal D = 4D.

According to the Lorentz's transformation, equation of which may generalize for any D of dimensions, we have:

$$x_a = f_a\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right) \quad ; \qquad a = 1 \ldots D \tag{3}$$

We define v as $v = \left[\text{const}, -\frac{dx_2}{dx_1}, \ldots, -\frac{dx_D}{dx_1}\right]$

Next

$$1 - \frac{v^2}{c^2} = p^2$$

If v is real, then p^2 is real, which corresponds with x^2 of quaternions with $x_o=0$ and for quaternions it is then and only then when $|v|>c$, because $x^2<0$ for $x_o=0$ (see (1)). In the case of generalized quaternions the condition (4) is fulfilled for any v because $x^2<0$ or $x^2>0$ according to the numbers n,m (see (2)). So generalized quaternions are better for our considerations giving them more general character.

So, for each dimension, we have quaternions and generalized quaternions with components x_1, x_2, x_3.

On the other hand $\sqrt{1-\frac{v^2}{c^2}}$ is a real number for $v<c$, so x_o appears, too. This way we have multiplication of 4 of each dimension. If v is complex then p^2 is complex and one quaternionic component and one real component appear.

2.3

The problem appears, what to do with N-nions which are not 2^n nions and which may be defined.

Any N-nion may be written with the shape:

$$x = \sum_{i=0}^{N-1} a_i e_i$$

$i \in N \cup \{0\}$

$a_i \in R$, e_i versors

$e_0^2 = 1$; $e_i^2 = -1$ for $i>0$

$e_0 e_i = e_i e_0 = e_i$

For $i>0$, $e_i e_j = e_i e_{i+l} =: e_{i+2l}$ where $l=j-i$, $j>i$, $i\ N-1$, $j \leq N-1$, $i<j$

$e_{i+2l} = e_{i+2l}$ if $i+2l \leq N-1$

$e_{i+2l} = e_{y \bmod N-1}$ if $i+2l > N-1$ (x)

This definition of multiplication of versors means, that if difference of two indices of versors is equal 1, the product of versor is the next versor. If the difference of two indices of versor is equal 2, the product of versors is the second next versor, and when this difference is equal l, the product of versor is l-th next versor (in comparison to multiplied versor with higher index).

The description (x) means cyclicity, which means that we take i+2l-th versor, as if we took the sequence once more after the end of the sequence of versors.
It uses only 2^n-nions (where $n \geq 2$, $n \in N$) for the description of dimensions of the Universe. The dimension is described by a number.
If we have m dimensions, all they are relativistic. It means that the object may move with velocity $|v|>c$ along the axis of each dimension, what means - according to Lorentz's transformation and geometric interpretation of complex numbers - the doubling of each dimension and so the doubling of all dimensions. As we start with m=4, so in this way we use only 2^n-nions although we may define any m-nions.

2.4

If v is quaternion then p^2 and x_a are quaternions (see (3) &

(4) so it is necessary to multiply by 4 each of D dimensions. It means that D = 4, 1o, 11, 26 ought to be replaced by

D = 16, 40, 44, 104

and the spaces with smaller number of dimensions may be subspaces of spaces with bigger number of dimensions or may penetrate them.

2.5

Let's consider facts supporting the idea that number 4 plays special role at the estimation of the number of dimensions of the Universe.

The first argument is beauty, essence and meaning of general relativity, the first formulation of which was 4 dimensional.

The second argument is 4 dimensional theory of Ashtekar's spacetime woven into loops.

The third argument is identical shape of real part of quaternion (and generalized quaternion) and spacetime interval.

The fourth argument is the fact that T duality of S duality is S duality an T duality in the case of 4 dimensions (according to M.J.Duff).

The fifth argument are considerations in §2.2 and §2.3.

But here the doubt arises. The analogical multiplication table may be constructed for octanions, 16-nions, 32-nions, generally 2^n-nions (n 2, n N) and so the infinite number of dimensions appears.

Solution of this problem may be the observation that Kaluza's-Klein's 5th dimension which is rolled into the loop doesn't differ from the A.Ashtekar's and J.Bars' loop [12]. Seemingly it is the next complication but it is clue to order these problems.

The reconcilement of enlargement of the number of dimensions of the Universe with strong tendences to reduction of these dimensions may be made as follows:

The primary time dimension of Superuniverse, along which our Universe moved before the Big Bang splits in the region of our Universe into 4 superdimensions, because - according to the principle of quantisation of gravitation - the dimension of Dirac's matrix must be equal to the dimension of

metric tensor; and the least number fulfilling this condition for Dirac's matrices is number four.
The number two would be right for massless Universe, in which Dirac's equation is exchanged for Weyl's equation, but our Universe and the Superuniverse have nonzero mass.
Because of the same reason, each of D=26, D=11 dimensions is splitting.
The single primary dimension had been splitted into 4 dimensions because of phenomenon of motion, because the velocity in the sense $v = \frac{dx}{dt}$ necessary to Lorentz's transformation, wouldn't be defined. This velocity is necessary as the result of the postulate that Maxwell's equation ought to have identical form in each coordinate system.

The basic number of dimensions of the Universe is number 4. The Universe has four superdimensions. We assume multiplicity of Universes and multibubble model of reality, one of bubbles of which is the Universe.
Each of superdimensions has D = 4, 1o, 11 or 26 dimensions, according to the type of Universe.
Simultaneously subdimensions exist having character of loops (the usual dimensions may be loop-like, too). The Ashtekar's and Bars' loops weawing D dimensional space may be these subdimensions.

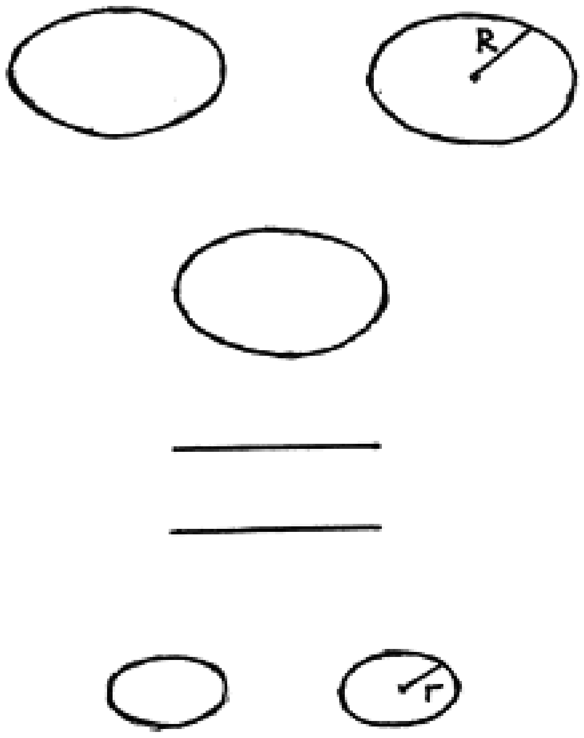
This way we have explained multiplication table for 2^n-nions and the analogy loop - Kaluza's-Klein's dimension, as well as the great significance of number 4.

The less the radius of loop, the greater nion is, which the loop is described by. So:

$$R \sim \frac{1}{2^n}$$

2.6

The division into 2^n dimensions is realized in two stages. At the beginning the twin dimension with equal perimeter arises and then it is divided into 2^n equal parts (which arise from both objects).



These parts are pasted into two-fold smaller parts and all two-stage process repeats n-times.

The fact htah the lenght of perimeter is conserved, implicates:

$O = 2\pi R$ (primary perimeter)

and

$O' = 2\pi r$ (perimeter after division)

Naturally:

$$O' = \frac{2\pi R}{2^n}$$

From the comparison we obtain:

$$r = \frac{R}{2^n}$$

$r' = 2^n R$ in the case of graving loop. We obtain it from the condition:

$$r\, r' = R^2$$

(compare 4.1)

2.7

The nature of superdimensions, subdimensions and loops is the same, what the rolling into the loop testifies, too; or the fact, that such fragments of 4-dimensional spacetime with such great curvature exist that one cannot get out of them. It would be superloops. These facts concern both dimensions and superdimensions.

The objects may obtain the character of dimensions [13].

And so the rolled string is loop, so it is quantisation. The wormhole falling off the black hole may be blind. In this case its end may roll into the loop.

Naturally 4-dimensional Einstein's equation concerns 4 dimensional space, too. It is true in the case of any dimensional space (D or D=4D), too, because it may be generalized for any number of dimensions [10].

2.8

Naturally, this problem may be analysed differently. We have for example D=11 dimensions so 11 cordinates. Each of them is described by quaternion, so we have 44 dimensions in traditional meaning.
Differently speaking, wh have 11 dimensions, each described by quaternion, and each describes its 4-dimensional space.
Analogically 26 dimensions are 26 coordinates described by quaternions, so 104 dimensions.
Each dimension of the Universe is splitted into dimensions an subdimensions, according to the formula:

$$S = 4 \cdot 2^n \quad n = 0, 1, 2...$$

The number of dimensions and subdimensions of the Universe is equal:

$$D = D \cdot 4 \cdot 2^n \qquad (5)$$

$n = 0$ - dimensions

$n \geqslant 1$ - subdimensions

Goldstone's equation [8] is equation for D and not for D.
Each dimension is 4-splitted, like the single primary dimension had been 4-splitted.
Equation (5) describes both splitting quaternion and splitting complex connected with $v > c$ and v complex.
The 4-splitting is more fundamental than 2-splitting, because 2-splitting is for $v > c$ and v complex, and 4-splitting is for $v < c$ and v quaternion and contains the case of real numbers in Lorentz's transformation and the case of complex numbers, too. The 4-splitting is enough to contain all more important cases. So, higher splittings are treated as subdimensions.

2.9

In any number of dimensions only one is time-like. This is this dimension which is used to obtain the derivatives of all remaining dimensions in purpose to obtain the vector of velocity.
We see only one of four superdimensions in the shape of D=11, 26 dimensions. The one dimension which we see, is equivalent

of time superdimension, which was split into D dimensions (one of which is time-like). It is characteristic superdimension, favoured as one of four, so we see it.
The fact, that we feel the presence of only one of D dimensions is implicated by the fact that this is reflection of four superdimensions.

III
The numbers of dimensions

3.1

Many examples supporting the idea, that the existing objects can be described by different, interesting number of dimensions, appear in the scientific literature.
And so 16-spinors correspond with 4-dimensional quaternion space. 16 dimensions of spinor correspond with 16 dimensional Dirac's matrices and the latter - using quantum-gravitational Dirac's-Einstein's equation - corresponds with 16-dimensional metric tensor [10].

3.2

It is necessary to add new variables in the nature of superdimensions to D=1 dimensional supermembrane, in purpose to explain the soliton quantum number [14].
There are super-p-branes which exist in bigger number of dimensions than D=1o and D=11.

3.3

The membranes display critical number of dimensions D=11.
There aren't critical numbers of dimensions for other structures. Because of taking under consideration the quaternions the number D=11 may be multiplied by 4 and one obtains the number D=44. Each number of the set $D \in \{11,12...26\}$ may be multiplied in this way. The set of numbers $D \in \{44,104\}$ corresponds with them.
There are two numbers being candidates for the number of dimensions of the Universe: D=11 and D=26 and in the quaternion version D=44 and D=104.
Generalizing the principle mass = α charge, we have hypermatter = $\alpha \cdot$ hypercharge [10,15].

20 multiplets of hypermatter correspond with 20 dimensions. We have here the reduction of the number of dimensions, an increase of the number of dimensions or the quaternion version of the number D=5 dimensions.

3.4

The charge quantum numbers of elementary string excitations are characterized by 30-dimensional vector $\alpha \in \Lambda_{30}$.
We have 32-dimensional lattice

$$\begin{pmatrix} \beta \\ m_1 \\ m_2 \end{pmatrix} \beta \in \Lambda_{30} \; ; \; m_1, m_2 \in \mathcal{E}$$

Λ_{30} is the subspace of the 32-dimensional space [16,17]
Dirac's matrices are 32x32 so $D_{eff}=32$ on the ground of the Dirac's-Einstein's equation. The group SO(32) corresponds with 32 colours and 32 dimensions.

3.5

Transformation O(6) x O(32) turns each vector around 22 dimensions and 6-dimensional vector. The charge vector exists in 22x6 dimensions [18].
Next, in the works 19,20 22x24 dimensional M-matrix exists corresponding - on the ground of Dirac's-Einstein's equation - - with 22x24 dimensions.

3.6

There is the confirmation that D=11 and D=44 play the main role in the description of the number of dimensions of the Universe.
In the already cited E.Cremmer's and S.Ferrara's work 11-bein $e_{\mu}^{+}(x)$ corresponds with 11 dimensions and 44-bein corresponds with 44 dimensions.

Spontaneous symmetry breaking is accompanied by the generation of superparticle (or, more generally, superobject). The central charge is responsible for supersymmetry breaking.
Supersymmetry may be broken, which breaks the condition for maximal dimension D=11.

In the Strominger's solitonic five-membrane solution half of supersymmetry is broken [22].
The same situation occurs in the case of elementary five-membrane.

IV
Growing and vanishing of loops

4.1

Loops are basic cells of the Universe. The expansion of loops is given by the formula [23] :

$$\text{Ex loop} = \sum_n b_n e^{n\phi} + \sum_n c_n e^{-n\phi} \qquad (6)$$

n positive integer (may be not integer)
ϕ scalar field characterizing dilaton. One can give it dimension-like sense. We introduce the transformation

$\phi \longrightarrow r$

These two terms testimony to the expansion of growing and vanishing loop.
In the first case we have the D-dimensional Universe and loops $r \sim \frac{1}{2^n}$ are subdimensions. The D-dimensional Universe is a supersphere containing subdimensions.

In the second case, loops $r \sim 2^n$ overgrow the D-dimensional Universe and themselves become a supersphere; and the D-dimensional Universe conserves the central position, being the nucleus of the supersphere.
The superposition of both cases is possible.

The situation in which loop dimensions correspond with the term $e^{n\phi}$ (what means that these loop dimensions grow and don't vanish and dominate over the D=44 or D=104 Universe - which is only a nucleus) ought to be treated seriously.
Naturally, subdimensions connected with the term $e^{-n\phi}$ vanish, creating loop structure which fills the space with smaller

and smaller meshes – smaller and smaller partition, till the infinity.
The fact that $e^{n\phi}$ means growing and $e^{-n\phi}$ means vanishing of loops, is a convention. The fact is important that both for positive ϕ and negative ϕ there are growing and vanishing members.
What is more important, these both terms exist simultaneously creating the structure of $(\text{super})^{k}_{k\to\infty}$ Universes and the structure D of D-dimensional space with smaller and smaller sublattices composed of smaller and smaller meshes.

4.2

The situation when D=44 or D=104-dimensional space is superstructure and loops $d = \frac{1}{2^m}$ $m>0$ and radius $r \sim \frac{1}{2^m}$, corresponds with the situation when loops have dimension $d = 2^m$ and radius $r \sim 2^m$ and create superstructure and substructure (nucleus) which is D=44 or D=104-dimensional space.
Both these sorts of the Universe are T dual and therefore really exist, because T duality means that the Universe looks in the small scale like in the great one. The infinite number of loop subdimensions may correspond with the infinite number of dimensions of the Hilbert's space.
The infinite dimensional Wilson's loops in QCD correspond with loops characterizing dimensions $d=2^n$ $n\to\infty$. It is simultaneously the bridge between the string-loop-membrane theory and QCD.

4.3

The space, although is woven into loops, is continuous.
Let's assume that the greatest resolution is equal ε. Radius of loop $r = \frac{1}{2^N}$ corresponds with it. But the infinite number $n > N$ exists, corresponding with denser and denser loop below the resolution. This argumentation may be repeated for any resolution ε. The infinite number of sublattices exists for any loop lattice, weaving continuous space.

4.4

The question arises, if the system of loops isn't distinguished reference system, which would be discrepant with Relativity. It is not so.
The rest mass of loop $m_o = 0$, which means that loops - similarly as photons - don't rest in any reference system and none distinguished reference system is connected with loops. In each reference system the velocity of loop $v = c$ and none of reference systems can be distinguished.

The remark that $m_o = 0$ removes one more unconsequence of this theory. Namely we avoid the following unconsequence, that mass not only of the whole system but of each mesh, would be infinite.

Loops fulfilling the space don't attenuate constant motions, because loops are marked by superfluidity, just like loops of rotons. Rotons move around loops [24] , loops stetch superfluid lattice.
The author is convinced that superfluid lattice exists in reality.

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